If $p=3$, $q=2$, then $n=\frac{m}{m^2-30}$, $=1$ when $m=6$; $\therefore$ $x=13$, $y=11$, and the numbers are 168 and 120.

If $p=4$, $q=1$, then $n=\frac{m}{m^2-60}$, $=2$ whem $m=8$; and then $x=34$, $y=31$, and the numbers are 1155 and 960.

PROBLEMS.

16. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

Find three numbers such that the cube of any one plus the sum of the squares of the other two will be a square.

17. Proposed by ARTEMAS MARTIN, LL. D., U. S Coast and Geodetic Survey Office, Washington, D. C.

Is it possible to find two positive whole numbers such that each of them, and also their sum and their difference, when *diminished* by unity shall all be squares?

Solutions to these problems should be received on or before December 1st.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

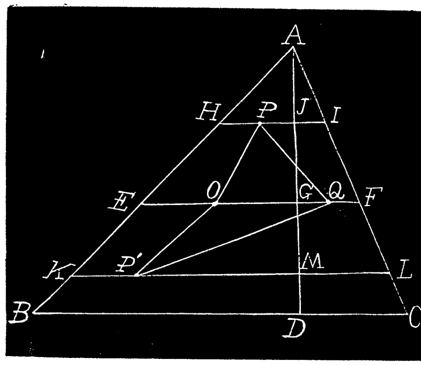
8. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Prove that the mean area of all triangles having their vertices upon the surface of a given triangle and bases parallel to the base of the given triangle, is $\frac{13}{210}$ (area of given triangle).

I. Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Represent AD by a, BC by b, and the area of $\triangle ABC$, $=\frac{1}{2}ab$, by $\triangle$.

Draw the random line EF, and on it choose at random the two points O and Q. Take any point P in the $\triangle\ AEF$, and complete the $\triangle\ OPQ$ the mean area of which is to be found. The point P may also be taken in the trapezoid $BCFE$, and then represented by P'. Put $AJ=y$, $AG=x$, $HP=z$, $EO=v$, $EQ=w$, $HI=n=(b\div a)\ y$, and $EF=m=(b\div a)x$; then will $GD=a-x$, $JG=x-y$, $GM=y-x$, $OQ=w-v$, $\triangle\ OPQ=\frac{1}{2}(x-y)(w-v)$, and $\Delta\ OP'Q=\frac{1}{2}(y-x)(w-v)$.



The required mean area, therefore, becomes

$$\Delta=\frac{\frac{1}{2}\int_0^a\int_0^m\int_0^w\left[\int_0^x\int_0^n(x-y)dydz+\int_x^a\int_0^n(y-x)dydz\right](w-v)dxdwdv}{\int_0^a\int_0^m\int_0^w\left[\int_0^x\int_0^n dydz+\int_x^a\int_0^n dydz\right]dxdwdv}$$

$$=\frac{1}{2}\left(\frac{12}{a^2b^3}\right)\left(\frac{b}{6a}\right)\int_0^a\int_0^m\int_0^w[2(a^3+x^3)-3a^2x](w-v)dxdwdv$$

$$=\frac{1}{2a^3b^2}\int_0^a\int_0^m[2(a^3+x^3)-3a^2x]dxw^2dw$$

$=\frac{b}{6a^6}\int_0^a[2(a^3+x^3)-3a^2x]dx=\frac{13}{210}\left(\frac{ab}{2}\right)=\frac{13}{210}\triangle$, which is the result given in the problem.

II. **Solution by the PROPOSER.**

Let ABC be the given triangle, MNP the triangle whose average area is required, having its base NP parallel to AB the base of the given triangle. Draw GMG' and CL parallel to AB.

Let $CH=u$, $CG=v$, $HN=x$, $HP=y$, $GM=z$, $GG'=z'$, $HH'=x'$. Then we have area $MNP=\frac{1}{2}(x-y)(u-v)\sin A$, when $v<u$

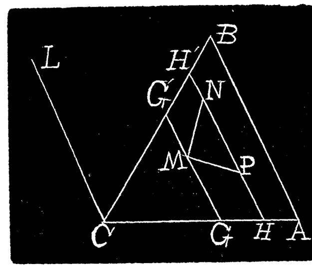
area $MNP=\frac{1}{2}(x-y)(v-u)\sin A$, when $v>u$.

The limits of u are 0 and b; of v, 0 and u, and u and b; of x, 0 and $x'=\frac{cu}{b}$; of y, 0 and x; of z, 0 and $z'=\frac{cu}{b}$.

Hence the required average area is

$$\Delta=\frac{\int_0^b\int_0^{x'}\int_0^x\left\{\int_0^u\int_0^{z'}\frac{1}{2}(x-y)(u-v)\sin A\ dvdz+\int_0^b\int_0^{z'}\frac{1}{2}(x-y)(v-u)\sin A\ dvdz\right\}dudxdy}{\int_0^b\int_0^{x'}\int_0^x\int_0^b\int_0^{z'}dudxdydvdz}$$

$$=\frac{6\sin A}{b^2c^3}\int_o^b\int_o^{x'}\int_o^x\left\{\int_o^u\int_o^{z'}(x-y)(u-v)dvdz\right.$$
$$\left.+\int_o^b\int_o^{z'}(x-y)(v-u)dvdz\right\}dudxdy$$
$$=\frac{6\sin A}{b^3c^2}\int_o^b\int_o^{x'}\int_o^x\left\{\int_o^u(x-y)(uv-v^2)dv\right.$$
$$\left.+\int_o^b(x-y)(v^2-uv)dv\right\}dudxdy$$
$$=\frac{\sin A}{b^3c^2}\int_o^b\int_o^{x'}\int_o^x(2u^3+2b^3-3b^2u)(x-y)dudxdy$$



$$=\frac{\sin A}{2b^3c^2}\int_o^b\int_o^{x'}(2u^3+2b^3-3b^2u)x^2dudx$$

$$=\frac{c\sin A}{6b^6}\int_o^b(2u^6+2b^3u^3-3b^2u^4)du$$

$$=\frac{13bc\sin A}{420}=\frac{13}{210}\tfrac{1}{2}(bc\sin A)=\frac{13}{210}\text{ (area of given triangle)}$$

9. **Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.**

Four numbers taken at random are multiplied together. What is the probability that the last digit will be 0?

I. **Solution by H. W. DRAUGHON, Clinton, Louisiana.**

The probability that the final digit will be odd is $(\frac{5}{10})^4=\frac{625}{10000}$; the probability that it will be 2, 4, 6, or 8, is $(\frac{4}{10})^4+4(\frac{4}{10})(\frac{4}{10})^3+6(\frac{4}{10})^2(\frac{4}{10})^2+4(\frac{4}{10})^3(\frac{4}{10})=\frac{15(4)^4}{10000}=\frac{3840}{10000}$. $\therefore$ the probability that it will be 0 is, $P=1-\frac{3840}{10000}-\frac{625}{10000}=\frac{5535}{10000}=\frac{1107}{2000}$.

II. **Solution by F P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

We know from *Hall and Knight's Higher Algebra* if n integers be taken at random and multiplied together, the probability that the last digit of the product is 1, 3, 7, or 9, is $P_1=\frac{4^n}{10^n}$; also, the probability that the last digit of the product is 2, 4, 6, or 8, is $P_2=\frac{8^n-4^n}{10^n}$; and, finally, the probability that the last digit of this product is 5, is $P_3=\frac{5^n-4^n}{10^n}$. Consequently the probability that the last digit of this product is *zero*, when $n=4$, is $P_4=1-(P_1+P_2+P_3)$; that is, $P_4=\frac{10^4-8^4-5^4+4^4}{10^4}=\frac{(5^4-4^4)(2^4-1)}{5^4\times2^4}=\frac{1107}{2000}$.

Solutions to this problem were also received from *Hon. JOSIAH DRUMMOND, P. H. PHILBRICK* and *J. F. W. SCHEFFER.*

PROBLEMS.

18. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**